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Adaptive control for nonlinear nonnegative dynamical systems

Wassim M. Haddad*, Tomohisa Hayakawa

School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150, USA

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Abstract

Nonnegative and compartmental models are widespread in engineering systems and life sciences and play a key role in the understanding of these systems. In this paper, we develop a direct adaptive control framework for nonlinear uncertain nonnegative and compartmental dynamical systems. The proposed framework is Lyapunov-based and guarantees partial asymptotic set-point regulation; that is, asymptotic set-point regulation with respect to part of the closed-loop system states associated with the plant. In addition, the adaptive controller guarantees that the physical system states remain in the nonnegative orthant of the state space.

Keywords: Adaptive control; Nonlinear nonnegative systems; Nonlinear compartmental systems; Set-point regulation

1. Introduction

Nonnegative and compartmental models are widespread in engineering systems and life sciences and play a key role in the understanding of these systems. Specifically, nonnegative and compartmental dynamical systems (Sandberg, 1978; Jacquez, 1985; Berman, Neumann, & Stern, 1989; Bernstein & Hyland, 1993; Jacquez & Simon, 1993; Farina & Rinaldi, 2000; Haddad, Chellaboina, & August, 2001) are composed of homogeneous interconnected subsystems (or compartments) which exchange variable nonnegative quantities of material with conservation laws describing transfer, accumulation, and elimination between the compartments and the environment. It thus follows from physical considerations that the state trajectory of such systems remains in the nonnegative orthant of the state space for nonnegative initial conditions. The range of applications of nonnegative and compartmental systems includes biological systems, chemical reaction systems, queuing systems, large-scale systems, ecological systems, economic systems, telecommunication systems, and power systems, to cite but a few examples (see Jacquez & Simon, 1993 and the numerous references therein). Using nonnegative and compartmental model structures, a Lyapunov-based direct adaptive control framework is developed that guarantees partial asymptotic set-point stability of the closed-loop system; that is, asymptotic set-point stability with respect to part of the closed-loop system states associated with the plant state variables. Furthermore, the remainder of the state associated with the adaptive controller gains is shown to be Lyapunov stable. In addition, adaptive controllers are constructed without requiring knowledge of the system dynamics while providing robust stabilization with respect to the nonnegative orthant. The framework developed in this paper is an extension of the adaptive control framework for linear nonnegative and compartmental dynamical systems developed in Haddad, Hayakawa, and Bailey (2003) to nonlinear systems.

2. Mathematical preliminaries

In this section we introduce notation, several definitions, and some key results concerning nonlinear nonnegative dynamical systems (Berman & Plemmons, 1979; Berman et al., 1989) that are necessary for developing the main results of this paper. Specifically, for $x \in \mathbb{R}^n$ we write $x \succeq 0$ (resp., $x \succ 0$) to indicate that every component of $x$ is nonnegative.
Definition 2.1. Let $T > 0$. A real function $u : [0, T] \rightarrow \mathbb{R}^n$ is a nonnegative (resp., positive) function if $u(t) \geq 0$ (resp., $u(t) > 0$) on the interval $[0, T]$.

Definition 2.2 (Berman & Plemmons, 1979). Let $A \in \mathbb{R}^{n \times n}$. $A$ is essentially nonnegative if $A_{(i,j)} \geq 0$, $i, j = 1, \ldots, n, i \neq j$.

The following definition and proposition are needed for the main results of the paper.

Definition 2.3. Let $f = [f_1, \ldots, f_d]^T : \mathcal{D} \rightarrow \mathbb{R}^n$, where $\mathcal{D}$ is an open subset of $\mathbb{R}^n$ that contains $\mathbb{R}^n_+$. Then $f$ is essentially nonnegative if $f_i(x) \geq 0$, for all $i = 1, \ldots, n$, and $x \in \mathbb{R}^n_+$ such that $x_i = 0$, where $x_i$ denotes the $i$th element of $x$.

For the next result, we consider the time-varying system

$$
\dot{x}(t) = f(x(t)) + G(x(t))u(t), \quad x(t_0) = x_0, \quad t \geq t_0,
$$

(1)

where $x(t) \in \mathbb{R}^n$, $t \geq t_0$, $u(t) \in \mathbb{R}^m$, $t \geq t_0$, $f : [t_0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ and satisfies $f(t,0) = 0$, $t \geq t_0$, and $G : \mathbb{R}^n \rightarrow \mathbb{R}^n \times m$.

Proposition 2.1 (Haddad et al., 2001). Consider the time-varying dynamical system (1) where $f(t,\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is Lipschitz continuous on $\mathbb{R}^n$ for all $t \in [t_0, \infty)$ and $f(\cdot,x) : [t_0, \infty) \rightarrow \mathbb{R}^n$ is continuous on $[t_0, \infty)$ for all $x \in \mathbb{R}^n$. If for every $t \in [0, \infty)$, $f(t,\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is essentially nonnegative and $G : \mathbb{R}^n \rightarrow \mathbb{R}^n \times m$ is nonnegative, then the solution $x(t), t \geq t_0$, to (1) is nonnegative for all $x_0 \in \mathbb{R}^n_+$.

3. Adaptive control for nonlinear nonnegative uncertain dynamical systems

In this section we consider the problem of characterizing adaptive feedback control laws for nonlinear nonnegative and compartmental uncertain dynamical systems to achieve set-point regulation in the nonnegative orthant. Specifically, consider the following controlled nonlinear uncertain system $\mathcal{G}$ given by

$$
\dot{x}(t) = f(x(t)) + G(x(t))u(t), \quad x(0) = x_0, \quad t \geq 0,
$$

(2)

where $x(t) \in \mathbb{R}^n$, $t \geq 0$, is the state vector, $u(t) \in \mathbb{R}^m$, $t \geq 0$, is the control input, $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is an unknown essentially nonnegative function and satisfies $f(0) = 0$, and $G : \mathbb{R}^n \rightarrow \mathbb{R}^n \times m$ is an unknown input matrix function. The control input $u(\cdot)$ in (2) is restricted to the class of admissible controls consisting of measurable functions such that $u(t) \in \mathbb{R}^m$, $t \geq 0$.

As discussed in the introduction, it follows from physical considerations that the state trajectories of nonnegative and compartmental dynamical systems remain in the nonnegative orthant of the state space for nonnegative initial conditions. Hence, in this paper we design adaptive controllers that guarantee that the controlled system states remain in the nonnegative orthant and converge to a desired equilibrium state. Specifically, for a given desired set point $x_0 \in \mathbb{R}^n_+$, our aim is to design a control input $u(t), t \geq 0$, such that $\lim_{t \rightarrow \infty} \|x(t) - x_0\| = 0$. We assume that we have $m$ control inputs and the input matrix function is given by

$$
G(x) = \begin{bmatrix}
B_x G_a(x) \\
B_0 (n-m) \times m
\end{bmatrix},
$$

(3)

where $B_x = diag[b_1, \ldots, b_m]$ is an unknown diagonal matrix and $G_a : \mathbb{R}^n \rightarrow \mathbb{R}^n \times m$ is a known nonnegative matrix function such that $\det G_a(x) \neq 0, x \in \mathbb{R}^n$. Furthermore, for the nonlinear system $\mathcal{G}$ we assume that the required properties for the existence and uniqueness of solutions are satisfied; that is, $f(\cdot)$, $G(\cdot)$, and $u(\cdot)$ satisfy sufficient regularity conditions such that (2) has a unique solution forward in time.

Theorem 3.1. Consider the nonlinear uncertain system $\mathcal{G}$ given by (2) where $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is essentially nonnegative and $G : \mathbb{R}^n \rightarrow \mathbb{R}^n \times m$ is given by (3). For a given $x_0 \in \mathbb{R}^n_+$, assume there exists a vector $u_c \in \mathbb{R}^m$ such that $f(x_c) \leq 0$ and

$$
0 = f(x_c) + \dot{B} u_c,
$$

(4)

where $\dot{B} \triangleq [B_a, 0_{m \times (n-m)}]^T$. Furthermore, assume that $b_1$ is unknown but $\text{sgn} b_i$ is known for all $i = 1, \ldots, m$, and assume there exist a rectangular block-diagonal matrix $K_x \triangleq \text{block-diag}[k_{1,1}^T, \ldots, k_{m,1}^T]$, where $k_{i,j} \in \mathbb{R}^n$ is such that $(\text{sgn} b_i) k_{i,j} \leq 0$, $i = 1, \ldots, m$, continuously differentiable functions $V : \mathbb{R}^n \rightarrow \mathbb{R}$, $i = 1, \ldots, m$, and $\dot{V}_i : \mathbb{R}^{n-m} \rightarrow \mathbb{R}$, and continuous functions $\ell : \mathbb{R}^n \rightarrow \mathbb{R}$ and $F_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $i = 1, \ldots, m$, with $F_i(x - x_c) \leq 0$ whenever $x_0 = 0$ and $F_i(0) = 0$, $i = 1, \ldots, m$, such that $V_i(\cdot)$ is positive definite, radially unbounded, $V_i(0) = 0$, $\ell(0) = 0$, and, for all $x \in \mathbb{R}^n$,

$$
V_i(\cdot) F_i(\cdot) \geq 0, \quad i = 1, \ldots, m,
$$

(5)

$$
0 = V_i'(\cdot) [f_i(\cdot) + \dot{B} K_x F_i(\cdot)] + \ell^T(\cdot) \ell(\cdot),
$$

(6)
where 
\[ V_s(e) = V_{s1}(e_1) + \cdots + V_{sn}(e_n) + \hat{V}_s(e_{m+1}, \ldots, e_n), \]
(7)
\[ f_s(e) \triangleq f(e+x_c) - f(x_c), \text{ and } F(e) \triangleq [F^T_{s1}(e), \ldots, F^T_{sm}(e)]^T. \]
Finally, let \( q_i \) and \( \hat{q}_i \), \( i = 1, \ldots, m \), be positive constants. Then the adaptive feedback control law
\[ u(t) = G_n^{-1}(x(t))K(t)F(x(t) - x_c) \]
\[ + G_n^{-1}(x(t))\phi(t), \]
(8)
where \( K(t) \triangleq \text{block-diag} [k_1^T(t), \ldots, k_m^T(t)] \), \( k_i(t) \in \mathbb{R}^e, i = 1, \ldots, m, t \geq 0 \), and \( \phi(t) \in \mathbb{R}^m, t \geq 0 \), with update laws
\[ k_i^T(t) = -(\text{sgn} b_i)\frac{q_i}{2} V'_s(x_i(t) - x_c)F_i^T(x(t) - x_c), \]
\[ i = 1, \ldots, m, \]
(9)
\[ \dot{\phi}_i(t) = \begin{cases} 
0 & \text{if } \phi_i(t) = 0 \text{ and} \\
V'_s(x_i(t) - x_c) & \text{otherwise,} 
\end{cases} \]
\[ i = 1, \ldots, m, \]
(10)
where \( k_i(0) \) and \( \phi_i(0) \) are such that \( (\text{sgn} b_i)k_i(0) \leq 0 \) and \( (\text{sgn} b_i)\phi_i(0) \geq 0, i = 1, \ldots, m \), guarantees that the solution \((x(t), K(t), \phi(t)) \equiv (x_0, K_0, \phi_0) \) of the closed-loop system given by \((2), (8)-(10)\) is Lyapunov stable. If, in addition, \( \ell^T(e)\ell(e) > m, e \in \mathbb{R}^m, e \neq 0 \), then \( x(t) \to x_c \) as \( t \to \infty \) for all \( x_0 \in \mathbb{R}^m_+ \). Furthermore, \( x(t) \geq 0, t \geq 0, \) for all \( x_0 \in \mathbb{R}^m_+ \).

**Proof.** Let \( e(t) \triangleq x(t) - x_c \) and note that with \( u(t), t \geq 0 \), given by \((8)\) it follows from \((2)\) that
\[ \dot{x}(t) = f(x(t)) + G(x(t))G_n^{-1}(x(t))K(t)F(x(t) - x_c) \]
\[ + G(x(t))G_n^{-1}(x(t))\phi(t), x(0) = x_0, t \geq 0, \]
(11)
or, equivalently, using \((3)\) and \((4)\),
\[ \dot{e}(t) = f_s(e(t)) + f(x_c) + \dot{\hat{B}}K_0F(e(t)) + F_i^T(x(t) - x_c) + \dot{\hat{B}}\phi(t) \]
\[ + \dot{\hat{B}}(K(t) - K_0)F(x(t) - x_c) + \dot{\hat{B}}\phi(t) - u_e(t), e(0) = x_0 - x_c, t \geq 0, \]
(12)
where \( f_s(e) \triangleq f_s(e) + B_0F(e) \). To show Lyapunov stability of the closed-loop system \((9), (10), \) and \((12)\) consider the Lyapunov function candidate
\[ V(e, K, \phi) = V_s(e) + \text{tr} (K - K_0)^T Q^{-1}(K - K_0) \]
\[ + (\phi - u_e)^T \hat{Q}^{-1}(\phi - u_e), \]
(13)
or, equivalently,
\[ V(e, K, \phi) = V_s(e) + m \sum_{i=1}^{m} \frac{|b_i|}{q_i} (k_i - k_{Gi})^T(k_i - k_{Gi}) \]
\[ + \sum_{i=1}^{m} \frac{|b_i|}{q_i} (\phi_i - u_{ei})^2, \]
where \( Q = \text{diag}[q_1/|b_1|, \ldots, q_m/|b_m|] \) and \( \hat{Q} = \text{diag}[\hat{q}_1/|b_1|, \ldots, \hat{q}_m/|b_m|] \). Note that \( F(0, K_0, u_e) = 0 \) and, since \( V_s(\cdot), Q, \) and \( \hat{Q} \) are positive definite, \( V(e, K, \phi) > 0 \) for all \((e, K, \phi) \neq (0, K_0, u_e) \). Furthermore, \( V(e, K, \phi) \) is radially unbounded. Now, letting \( e(t), t \geq 0 \), denote the solution to \((12)\) and using \((9)\) and \((10)\), it follows that the Lyapunov derivative along the closed-loop system trajectories is given by
\[
\dot{V}(e(t), K(t), \phi(t)) = V_s'(e(t))[f_s(e(t)) + \dot{\hat{B}}(K(t))
\]
\[ - K_0]F(x(t) - x_c) + \dot{\hat{B}}(\phi(t) - u_e)]
\[ + m \sum_{i=1}^{m} \frac{2|b_i|}{q_i} k_i^T(t)(k_i(t) - k_{Gi}) \]
\[ + \sum_{i=1}^{m} \frac{2|b_i|}{q_i} (\phi_i(t) - u_{ei})\phi_i(t)
\[ = - \ell^T(e(t))\ell(e(t))
\[ + m \sum_{i=1}^{m} V'_s(e_i(t))b_i(k_i(t) - k_{Gi})^TF_i(e(t)) \]
\[ + \sum_{i=1}^{m} b_i V'_s(e_i(t))(\phi_i(t) - u_{ei}) \]
\[ + \sum_{i=1}^{m} \frac{2|b_i|}{q_i} (\phi_i(t) - u_{ei})\phi_i(t)
\[ = - \ell^T(e(t))\ell(e(t)) + \sum_{i=1}^{m} (\phi_i(t) - u_{ei})
\[ \times \left[ b_i V'_s(e_i(t)) + \frac{2|b_i|}{q_i} \phi_i(t) \right], \]
(14)
where in \((14)\) we used \( |b_i|/\text{sgn} b_i = b_i \). Now, for each \( i \in \{1, \ldots, m\} \) and for the two cases given in \((10)\), the last term on the right-hand side of \((14)\) gives:

(i) If \( \phi_i(t) = 0 \) and \( V'_s(x_i(t) - x_c) \geq 0 \), then \( \phi_i(t) = 0 \) and hence, since, using \((4)\), \( b_i u_{ei} \geq 0 \), it follows that
\[
(\phi_i(t) - u_{ei}) \left[ b_i V'_s(e_i(t)) + \frac{2|b_i|}{q_i} \phi_i(t) \right]
\[ = - b_i u_{ei} V'_s(x_i(t) - x_c) \leq 0. \]
(ii) Otherwise, \( \dot{\phi}(t) = -(\text{sgn} \, b_i) \dot{q}_i / 2 V'_e(x(t) - x_c) \) and hence

\[
\left( \dot{\phi}(t) - u_c \right) \begin{bmatrix} b_i V'_e(x(t)) + \frac{2|b_i|}{q_i} \dot{\phi}(t) \end{bmatrix} = 0.
\]

Hence, it follows that in either case

\[
\dot{V}(e(t), K(t), \phi(t)) \leq -l^T(e(t))f(e(t)) \leq 0, \quad t \geq 0,
\]

which proves that the solution \( (e(t), K(t), \phi(t)) \equiv (0, K_y, u_c) \) to \( (9), (10), \) and \( (12) \) is Lyapunov stable. Furthermore, it follows from Theorem 2 of Chellaboina and Haddad (2002) that \( f(e(t)) \rightarrow 0 \) as \( t \rightarrow \infty. \) If, in addition, \( f^T(e(t))f(e(t)) > 0, \) \( e \in \mathbb{R}^n, e \neq 0, \) then \( x(t) \rightarrow x_c \) as \( t \rightarrow \infty \) for all \( x_0 \in \mathbb{R}^n. \)

Finally, to show that \( x(t) \not\rightarrow 0, t \geq 0, \) for all \( x_0 \in \mathbb{R}^n \) note that the closed-loop system \( (2), (8)-(10) \) is given by

\[
\dot{x}(t) = f(x(t)) + \tilde{B}K(x(t) - x_c) + \tilde{B}\phi(t)
\]

\[
= \tilde{f}(t,x(t)) + \tilde{B}K(x(t) - x_c) + \tilde{B}\phi(t)
\]

\[
= \dot{f}(t,x(t)) + v(t) + w(t), \quad x(0) = x_0, \quad t \geq 0,
\]

where \( \tilde{f}(x - x_c) = [\tilde{F}^T_1(x - x_c), \ldots, \tilde{F}^T_m(x - x_c)]^T, \)

\[
\tilde{F}_i(x - x_c) = F_i(x - x_c)_{|x_0| = 0}, \quad i = 1, \ldots, m,
\]

\[
\tilde{f}(t,x) \triangleq f(x) + \tilde{B}K(t)[F(x - x_c) - \tilde{F}(x - x_c)],
\]

\[
v(t) \triangleq \begin{bmatrix} b_1 k_1^T(t) \tilde{F}_1(x(t) - x_c) \\ \vdots \\ b_m k_m^T(t) \tilde{F}_m(x(t) - x_c) \end{bmatrix}
\]

\[
w(t) \triangleq \begin{bmatrix} b_1 \phi_1(t) \\ \vdots \\ b_m \phi_m(t) \end{bmatrix}.
\]

Now, since, by \( (5), (9), \) and \( (10), \) \( (\text{sgn} \, b_i) k_i^T(t) \leq 0, \)

\( i = 1, \ldots, m,\) and \( (\text{sgn} \, b_i) \phi_i(t) \geq 0, \) \( t \geq 0, i = 1, \ldots, m,\)

and since \( \tilde{F}_i(x(t) - x_c) \leq 0, \) \( t \geq 0, i = 1, \ldots, m,\) it follows that for every \( t \in [0, \infty), \) \( f(t,x(t)) \) is essentially nonnegative, \( v(t) \geq 0, \) and \( w(t) \geq 0. \) Hence, it follows from Proposition 2.1 that \( x(t) \geq 0, t \geq 0, \) for all \( x_0 \in \mathbb{R}^n. \)
\[ \dot{x}_1(t) = -a_{21}(x_1(t))x_1(t) + a_{12}(x_1(t))x_2(t) + bu(t), \]
\[ x_1(0) = x_{10}, \quad t \geq 0, \quad \text{(30)} \]
\[ \dot{x}_2(t) = a_{21}(x_1(t))x_1(t) - a_{12}(x_1(t))x_2(t), \]
\[ x_2(0) = x_{20}, \quad \text{(31)} \]

where \(a_{21}(x_1) \triangleq c_1Q(x_1), a_{12}(x_1) \triangleq c_2 + c_3Q(x_1), Q(x_1) \triangleq 1/(c_4x_1 + c_5), \) and \(c_1, \ldots, c_5, \) and \(b \) are unknown positive constants. Note that with \(x = [x_1, x_2]^T, \) (30) and (31) can be written in the form of (2) with \(f(x) = [ -a_{21}(x_1)x_1 + a_{12}(x_1)x_2, a_{21}(x_1)x_1 - a_{12}(x_1)x_2]^T \) and \(Q(x) = B = [b, 0]^T. \) Here, our objective is to regulate \(x_1 \) around the desired value \(x_{10} \geq 0. \) Note that \(x_{20} = c_1Q(x_1)x_{10}/(c_2 + c_3Q(x_{10})) \) and \(u_0 = 0 \) satisfy the equilibrium condition (4) with \(x = [x_{10}, x_{20}]^T. \) Furthermore, define \(e(t) \triangleq x(t) - x_{10} \) so that \(f_e(e) \) is given by

\[ f_e(e) = \begin{bmatrix} -[a_{21}(e_1 + x_{10}) + a_{12}(e_1 + x_{10})e_2 + x_{20}] \\ -[ -a_{21}(x_{10}) + a_{12}(x_{10})e_2 ] \\ a_{21}(e_1 + x_{10})e_1 + a_{12}(e_1 + x_{10})e_2 + a_{12}(e_1 + x_{10})e_2 + x_{20} \\ -x_{20}[a_{21}(e_1 + x_{10}) - a_{12}(e_1 + x_{10})e_1] \\ x_{20}[a_{21}(e_1 + x_{10}) - a_{12}(e_1 + x_{10})e_2] + x_{20} \\ -x_{20}[a_{21}(e_1 + x_{10}) - a_{12}(e_1 + x_{10})e_2] + k_6e_2 \end{bmatrix}. \]

(32)

Furthermore, let \(K_g = k_g/b, F_1(e) = e_1, \) and \(V_g(e) = e_1^2 + e_2^2 \) so that \(V_0'(e)F_1(e) = 2e_1^2 \geq 0. \) Next, note that

\[ V_g'(e)[f_e(e) + \hat{BK}_gF_1(e)] = e_1[f_e(e) + k_6e_1] + e_2f_{c2}(e) = -[a_{21}(e_1 + x_{10}) + k_6(e_1x_{10})]e_1^2 + a_{12}(e_1 + x_{10})e_2 \\ -x_{20}[a_{21}(e_1 + x_{10}) - a_{12}(e_1 + x_{10})e_1]e_1 \\ -x_{20}[a_{21}(e_1 + x_{10}) - a_{12}(e_1 + x_{10})e_2]e_2 \\ + a_{21}(e_1 + x_{10})e_2 - a_{21}(e_1 + x_{10})e_2 + a_{12}(e_1 + x_{10})e_2 \\ + x_{20}[a_{21}(e_1 + x_{10}) - a_{12}(e_1 + x_{10})e_2]e_2 \\ -x_{20}[a_{21}(e_1 + x_{10}) - a_{12}(e_1 + x_{10})e_2]e_2 + k_6e_2 \]

(33)

where \(f_e(\cdot) \) denotes the \(i\)th component of \(f_e(\cdot), \quad i = 1, 2, \) and \(-k_6 \in \mathbb{R}_+ \). Now, since \(Q(\cdot) \) is Lipschitz continuous there exist positive constants \(x \) and \(q \) such that \( ||Q(e_1 + x_{10}) - Q(x_{10})|| \leq \epsilon x_1^2 \) and \( ||Q(e_1 + x_{10}) - Q(x_{10})|| \leq \beta ||e_1||e_2 || \), and hence it follows that

4. Illustrative numerical example

In this section we present a numerical example to demonstrate the utility of the proposed direct adaptive control framework. Specifically, consider the controlled two-compartment nonnegative dynamical system given by

\[ x_1(t) = \begin{bmatrix} A_{11}(\hat{e}) & A_{12} \\ A_{21} & A_{22} \end{bmatrix} e - \begin{bmatrix} A_{11}(\hat{e}) \\ 0 \end{bmatrix} e \]

\[ + \begin{bmatrix} B_u \\ 0_{(n-m)\times m} \end{bmatrix} \begin{bmatrix} -\hat{q}_1(\hat{e})/b_1 + (\text{sgn} \ b_1)\hat{e}_1 \\ \vdots \\ -\hat{q}_m(\hat{e})/b_m + (\text{sgn} \ b_m)\hat{e}_m \end{bmatrix} \]

\[ = (\hat{A} + \hat{B}\Phi C)e. \]

(24)

In this case, with \(V_0'(e) = e^TPe, \) the adaptive feedback controller (8) with update laws (9), (10) or, equivalently,

\[ \hat{k}_i(t) = -\text{sgn} \ b_i q_i(x_i(t) - x_{ei}), \quad i = 1, \ldots, m, \]

(25)

\[ \hat{\phi}_i(t) = \begin{cases} 0 & \text{if } \phi_i(t) = 0 \text{ and } \hat{\phi}_i(t) > 0, \\ -\text{sgn} \ b_i \hat{q}_i(x_i(t) - x_{ei}) & \text{otherwise}, \end{cases} \quad i = 1, \ldots, m, \]

(26)

where \(k_i(0)\) and \(\phi_i(0)\) are such that \((\text{sgn} \ b_i)k_i(0) \leq 0 \) and \((\text{sgn} \ b_i)\phi_i(0) \geq 0, i = 1, \ldots, m, \) with \(q_i \) and \(\hat{q}_i \) in (9) and (10) replaced by \(q_i/p_i \) and \(\hat{q}_i/p_i \) respectively, guarantees global asymptotic stability of the nonlinear uncertain dynamical system (2) with \(f(x) = f_e(e) + f(x_0), \) where \(f_e(e) \) satisfies (19).

It is important to note that the adaptive feedback controller (8) with update laws (25),(26) does not require knowledge of the system dynamics (19). All that is required is that \(A_{22} \) be asymptotically stable. In the case where \(A_{11}(\hat{e}) = 0 \) and \(G_0(x) = I_m, \) we can simply take \(F(\hat{e}) = \hat{e}. \) In this case, the adaptive feedback controller (8) with update laws (9), (10) collapses to

\[ u_i(t) = k_i(t)(x_i(t) - x_{ei}) + \phi_i(t), \quad i = 1, \ldots, m, \]

(27)

\[ \hat{k}_i(t) = -\text{sgn} \ b_i q_i(x_i(t) - x_{ei})^2, \quad i = 1, \ldots, m, \]

(28)

\[ \hat{\phi}_i(t) = \begin{cases} 0 & \text{if } \phi_i(t) = 0 \text{ and } \hat{\phi}_i(t) > 0, \\ -\text{sgn} \ b_i \hat{q}_i(x_i(t) - x_{ei}) & \text{otherwise}, \end{cases} \quad i = 1, \ldots, m, \]

(29)

where \(k_i(0)\) and \(\phi_i(0)\) are such that \((\text{sgn} \ b_i)k_i(0) \leq 0 \) and \((\text{sgn} \ b_i)\phi_i(0) \geq 0, i = 1, \ldots, m. \) This is precisely the result given in Haddad et al. (2003).
Now, it follows from Theorem 3.1 that with $c$ for any positive constant $c_1,\ldots, c_5$, and $h$. For $x_1 = 2$ and $x_1 = 2, c_3 = 0.1, c_3 = 3, c_4 = c_5 = 1, b = 3, q_1 = 0.01, \hat{q}_1 = 0.1$, and initial conditions $x(0) = [5,8]^T, k_1(0) = 0$, and $\phi_1(0) = 1$, Fig. 1 shows the state trajectories of the controlled system (30), (31) versus time. Finally, Fig. 2 shows the control signal and the adaptive gain history versus time.

5. Conclusion

Nonnegative and compartmental dynamical systems are widely used to capture system dynamics involving the interchange of mass and energy between homogeneous subsystems or compartments. In this paper, we developed an adaptive control framework for adaptive set-point regulation of nonlinear nonnegative and compartmental systems. Using Lyapunov methods the proposed framework was shown to guarantee partial asymptotic set-point stability of the closed-loop system while additionally guaranteeing the nonnegativity of the closed-loop system states associated with the plant dynamics.

References