Neural network hybrid adaptive control for nonlinear uncertain impulsive dynamical systems

Tomohisa Hayakawa\textsuperscript{a,1}, Wassim M. Haddad\textsuperscript{b,*}, Konstantin Y. Volyanskyy\textsuperscript{b,2}

\textsuperscript{a} Department of Mechanical and Environmental Informatics, Tokyo Institute of Technology, Tokyo 332-0012, Japan
\textsuperscript{b} School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150, United States

Received 30 October 2007; accepted 17 January 2008

Abstract

A neural network hybrid adaptive control framework for nonlinear uncertain hybrid dynamical systems is developed. The proposed hybrid adaptive control framework is Lyapunov-based and guarantees partial asymptotic stability of the closed-loop hybrid system; that is, asymptotic stability with respect to part of the closed-loop system states associated with the hybrid plant states. A numerical example is provided to demonstrate the efficacy of the proposed hybrid adaptive stabilization approach.

© 2008 Elsevier Ltd. All rights reserved.

Keywords: Hybrid adaptive control; Neural networks; Nonlinear hybrid systems; Impulsive dynamical systems; System uncertainty; Stabilization; Sector-bounded nonlinearities

1. Introduction

Modern complex engineering systems involve multiple modes of operation placing stringent demands on controller design and implementation of increasing complexity. Such systems typically possess a multiechelon hierarchical hybrid control architecture characterized by continuous-time dynamics at the lower levels of the hierarchy and discrete-time dynamics at the higher levels of the hierarchy (see [1–3] and the numerous references therein). The lower-level units directly interact with the dynamical system to be controlled while the higher-level units receive information from the lower-level units as inputs and provide (possibly discrete) output commands which serve to coordinate and reconcile the (sometimes competing) actions of the lower-level units. The hierarchical controller organization reduces processor cost and controller complexity by breaking up the processing task into relatively small pieces and decomposing the fast and slow control functions. Typically, the higher-level units perform logical checks that determine system mode operation, while the lower-level units execute continuous-variable commands for a given system mode of operation. The mathematical description of many of these systems can be characterized by impulsive differential equations [3–7].
The purpose of feedback control is to achieve desirable system performance in the face of system uncertainty. To this end, adaptive control along with robust control theory have been developed to address the problem of system uncertainty in control-system design. In contrast to fixed-gain robust controllers, which maintain specified constants within the feedback control law to sustain robust performance, adaptive controllers directly or indirectly adjust feedback gains to maintain closed-loop stability and improve performance in the face of system uncertainties. Specifically, indirect adaptive controllers utilize parameter update laws to identify unknown system parameters and adjust feedback gains to account for system variation, while direct adaptive controllers directly adjust the controller gains in response to plant variations. The inherent nonlinearities and system uncertainties in hierarchical hybrid control systems and the increasingly stringent performance requirements required for controlling such modern complex embedded systems necessitates the development of hybrid adaptive nonlinear control methodologies.

In a recent paper [8], a hybrid adaptive control framework for adaptive stabilization of multivariable nonlinear uncertain impulsive dynamical systems was developed. In particular, a Lyapunov-based hybrid adaptive control framework was developed that guarantees partial asymptotic stability of the closed-loop system; that is, asymptotic stability with respect to part of the closed-loop system states associated with the hybrid plant dynamics. Furthermore, the remainder of the state associated with the adaptive controller gains was shown to be Lyapunov stable. As is the case in the continuous and discrete-time adaptive control literature [9–13], the system errors in [8] are captured by a constant linearly parameterized uncertainty model of a known structure but unknown variation. This uncertainty characterization allows the system nonlinearities to be parameterized by a finite linear combination of basis functions within a class of function approximators such as rational functions, spline functions, radial basis functions, sigmoidal functions, and wavelets. However, this linear parametrization of basis functions cannot, in general, exactly capture the unknown system nonlinearity.

Neural network-based adaptive control algorithms have been extensively developed in the literature, wherein Lyapunov-like functions are used to ensure that the neural network controllers can guarantee ultimate boundedness of the closed-loop system states rather than closed-loop asymptotic stability. Ultimate boundedness ensures that the plant states converge to a neighborhood of the origin (see, for example, [14–16] for continuous-time systems and [17–19] for discrete-time systems). The reason why stability in the sense of Lyapunov is not guaranteed stems from the fact that the uncertainties in the system dynamics cannot be perfectly captured by neural networks using the universal function approximation property and the residual approximation error is characterized via a norm bound over a given compact set. Ultimate boundedness guarantees, however, are often conservative since standard Lyapunov-like theorems that are typically used to show ultimate boundedness of the closed-loop hybrid system states provide only sufficient conditions, while neural network controllers may possibly achieve plant state convergence to an equilibrium point.

In this paper, we develop a neural hybrid adaptive control framework for a class of nonlinear uncertain impulsive dynamical systems which ensures state convergence to a Lyapunov stable equilibrium as well as boundedness of the neural network weighting gains. Specifically, the proposed framework is Lyapunov-based and guarantees partial asymptotic stability of the closed-loop hybrid system; that is, Lyapunov stability of the overall closed-loop states and convergence of the plant state. The neuroadaptive controllers are constructed without requiring explicit knowledge of the hybrid system dynamics other than the fact that the plant dynamics are continuously differentiable and that the approximation error of the unknown system nonlinearities lies in a small gain-type norm bounded conic sector over a compact set. Hence, the overall neuroadaptive control framework captures the residual approximation error inherent in linear parameterizations of system uncertainty via basis functions. Furthermore, the proposed neuroadaptive control architecture is modular in the sense that if a nominal linear design model is available, then the neuroadaptive controller can be augmented to the nominal design to account for system nonlinearities and system uncertainty.

Finally, we emphasize that we do not impose any linear growth condition on the system resetting (discrete) dynamics. In the literature on classical (non-neural) adaptive control theory for discrete-time systems, it is typically assumed that the nonlinear system dynamics have the linear growth rate which is necessary in proving Lyapunov stability rather than practical stability (ultimate boundedness). Our novel characterization of the system uncertainties (i.e., the small gain-type bound on the norm of the modeling error) allows us to prove asymptotic stability without requiring a linear growth condition on the system dynamics.

2. Mathematical preliminaries

In this section, we introduce notation, definitions, and some key results concerning impulsive dynamical systems [3–7,20]. Let \( \mathbb{R} \) denote the set of real numbers, \( \mathbb{R}^n \) denote the set of \( n \times 1 \) real column vectors, \((\cdot)^T\) denote transpose,
denote the Moore–Penrose generalized inverse, \( N \) denote the set of nonnegative integers, \( \mathbb{N}^n \) (resp., \( \mathbb{P}^n \)) denote the set of \( n \times n \) nonnegative (resp., positive) definite matrices, and \( I_n \) denote the \( n \times n \) identity matrix. Furthermore, we write \( \text{tr}(\cdot) \) for the trace operator, \( \ln(\cdot) \) for the natural log operator, \( \lambda_{\min}(M) \) (resp., \( \lambda_{\max}(M) \)) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix \( M \), \( \sigma_{\max}(M) \) for the maximum singular value of the matrix \( M \), \( V'(x) \) for the Fréchet derivative of \( V \) at \( x \), and \( \text{dist}(p, M) \) for the smallest distance from a point \( p \) to any point in the set \( M \), that is, \( \text{dist}(p, M) = \inf_{x \in M} \| p - x \| \).

In this paper, we consider controlled state-dependent [3] impulsive dynamical systems \( G \) of the form

\[
\dot{x}(t) = f_c(x(t)) + G_c(x(t))u_c(t), \quad x(0) = x_0, \quad x(t) \not\in Z_x, \quad (1)
\]

\[
\Delta x(t) = f_d(x(t)) + G_d(x(t))u_d(t), \quad x(t) \in Z_x, \quad (2)
\]

where \( t \geq 0, x(t) \in D \subseteq \mathbb{R}^n, D \) is an open set with \( 0 \in D \), \( \Delta x(t) = x(t^+) - x(t) \), \( u_c(t) \in U_c \subseteq \mathbb{R}^{m_c}, u_d(t_k) \in U_d \subseteq \mathbb{R}^{m_d}, t_k \) denotes the \( k \)th instant of time at which \( x(t) \) intersects \( Z_x \) for a particular trajectory \( x(t) \), \( f_c : D \rightarrow \mathbb{R}^n \) is Lipschitz continuous and satisfies \( f_c(0) = 0 \), \( G_c : D \rightarrow \mathbb{R}^{n \times m_c}, f_d : Z_x \rightarrow \mathbb{R}^n \) is continuous, \( G_d : Z_x \rightarrow \mathbb{R}^{n \times m_d} \) is such that \( \text{rank} G_d(x) = m_d, x \in Z_x \), and \( Z_x \subset D \) is the resetting set. Here, we assume that \( u_c(\cdot) \) and \( u_d(\cdot) \) are restricted to the class of admissible inputs consisting of measurable functions such that \( (u_c(t), u_d(t_k)) \in U_c \times U_d \) for all \( t \geq 0 \) and \( k \in \mathcal{N}_{[0, t]} : \{ k : 0 \leq t_k < t \} \), where the constrained set \( U_c \times U_d \) is given with \( (0, 0) \in U_c \times U_d \). We refer to the differential equation (1) as the continuous-time dynamics, and we refer to the difference equation (2) as the resetting law.

The equations of motion for the closed-loop impulsive dynamical system (1) and (2) with hybrid adaptive feedback controllers \( u_c(\cdot) \) and \( u_d(\cdot) \) have the form

\[
\dot{x}(t) = \hat{f}_c(\bar{x}(t)), \quad \bar{x}(0) = \bar{x}_0, \quad \bar{x}(t) \not\in \bar{Z}_\varepsilon, \quad (3)
\]

\[
\Delta \bar{x}(t) = \hat{f}_d(\bar{x}(t)), \quad \bar{x}(t) \in \bar{Z}_\varepsilon, \quad (4)
\]

where \( \hat{f}_c : \bar{D} \rightarrow \mathbb{R}^\bar{n} \) and \( \hat{f}_d : \bar{D} \rightarrow \mathbb{R}^\bar{n} \) denote the closed-loop continuous-time and resetting dynamics, respectively, with \( \hat{f}_c(\bar{x}_e) = 0, \) where \( \bar{x}_e \in \bar{D} \setminus \bar{Z}_\varepsilon \) denotes the closed-loop equilibrium point, and \( \bar{n} \) denotes the dimension of the closed-loop system state. A function \( \bar{x} : T_{\bar{x}_0} \rightarrow \bar{D} \) is a solution to the impulsive system (3) and (4) on the interval \( T_{\bar{x}_0} \subseteq \mathbb{R} \) with initial condition \( \bar{x}(0) = \bar{x}_0 \), where \( T_{\bar{x}_0} \) denotes the maximal interval of existence of a solution to (3) and (4), if \( \bar{x}(\cdot) \) is left-continuous and \( \bar{x}(t) \) satisfies (3) and (4) for all \( t \in T_{\bar{x}_0} \). For further discussion on solutions to impulsive differential equations, see [3–6]. For convenience, we use the notation \( s(t, \bar{x}_0) \) to denote the solution \( \bar{x}(t) \) of (3) and (4) at time \( t \geq 0 \) with initial condition \( \bar{x}(0) = \bar{x}_0 \).

In this paper, we assume that Assumptions A1 and A2 established in [3,7] hold; that is, the resetting set is such that resetting removes \( \bar{x}(t_k) \) from the resetting set and no trajectory can intersect the interior of \( \bar{Z}_\varepsilon \). Hence, as shown in [3,7], the resetting times are well defined and distinct. Since the resetting times are well defined and distinct and since the solution to (3) exists and is unique it follows that the solution of the impulsive dynamical system (3) and (4) also exists and is unique over a forward time interval. However, it is important to note that the analysis of impulsive dynamical systems can be quite involved. In particular, such systems can exhibit Zenoness and beating as well as confluence, wherein solutions exhibit infinitely many resettings in a finite time, encounter the same resetting surface a finite or infinite number of times in zero time, and coincide after a certain point in time. In this paper we allow for the possibility of confluence and Zeno solutions; however, A2 precludes the possibility of beating. Furthermore, since not every bounded solution of an impulsive dynamical system over a forward time interval can be extended to infinity due to Zeno solutions, we assume that existence and uniqueness of solutions are satisfied in forward time. For details see [3].

Next, we provide a key result from [3,7,20] involving an invariant set stability theorem for hybrid dynamical systems. For the statement of this result the following key assumption is needed.

**Assumption 2.1** ([3,7,20]). Let \( s(t, \bar{x}_0), t \geq 0 \), denote the solution of (3) and (4) with initial condition \( \bar{x}_0 \in \bar{D} \). Then for every \( \bar{x}_0 \in \bar{D} \), there exists a dense subset \( T_{\bar{x}_0} \subseteq [0, \infty) \) such that \( [0, \infty) \setminus T_{\bar{x}_0} \) is (finitely or infinitely) countable and for every \( \epsilon > 0 \) and \( t \in T_{\bar{x}_0} \), there exists \( \delta(\epsilon, \bar{x}_0, t) > 0 \) such that if \( \| \bar{x}_0 - y \| < \delta(\epsilon, \bar{x}_0, t), \) \( y \in \bar{D} \), then \( \| s(t, \bar{x}_0) - s(t, y) \| < \epsilon \).
Assumption 2.1 is a generalization of the standard continuous dependence property for dynamical systems with continuous flows to dynamical systems with left-continuous flows. Specifically, by letting $\mathcal{T}_{\bar{x}_0} = \overline{\mathcal{T}_{x_0}} = [0, \infty)$, where $\overline{\mathcal{T}_{x_0}}$ denotes the closure of the set $\mathcal{T}_{x_0}$, Assumption 2.1 specializes to the classical continuous dependence of solutions of a given dynamical system with respect to the system’s initial conditions $\bar{x}_0 \in \mathcal{D}$ [21]. Since solutions of impulsive dynamical systems are not continuous in time and solutions are not continuous functions of the system initial conditions, Assumption 2.1 is needed to apply the hybrid invariance principle developed in [7,20] to hybrid adaptive systems. Henceforth, we assume that the hybrid adaptive feedback controllers $u_\ell(\cdot)$ and $u_d(\cdot)$ are such that closed-loop hybrid system (3) and (4) satisfies Assumption 2.1. Necessary and sufficient conditions that guarantee that the nonlinear impulsive dynamical system $\mathcal{G}$ satisfies Assumption 2.1 are given in [3,20]. A sufficient condition that guarantees that the trajectories of the closed-loop nonlinear impulsive dynamical system (3) and (4) satisfy Assumption 2.1 are Lipschitz continuity of $\bar{f}_c(\cdot)$ and the existence of a continuously differentiable function $\mathcal{X} : \mathcal{D} \to \mathbb{R}$ such that the resetting set is given by $\mathcal{Z}_\delta = \{ \bar{x} \in \mathcal{D} : \mathcal{X}(\bar{x}) = 0 \}$, where $\mathcal{X}'(\bar{x}) \neq 0$, $\bar{x} \in \mathcal{Z}_\delta$, and $\mathcal{X}'(\bar{x}) \bar{f}_c(\bar{x}) \neq 0$, $\bar{x} \in \mathcal{Z}_\delta$. The last condition above ensures that the solution of the closed-loop hybrid system is not tangent to the resetting set $\mathcal{Z}_\delta$ for all initial conditions $\bar{x}_0 \in \mathcal{D}$. For further discussion on Assumption 2.1, see [3,7,20].

The following theorem proven in [7,20] is needed to develop the main results of this paper.

Theorem 2.1 ([7,20]). Consider the nonlinear impulsive dynamical system $\mathcal{G}$ given by (3) and (4), assume $\mathcal{D}_c \subset \mathcal{D}$ is a compact positively invariant set with respect to (3) and (4), and assume that there exists a continuously differentiable function $V : \mathcal{D}_c \to \mathbb{R}$ such that

$$V'(\bar{x})\bar{f}_c(\bar{x}) \leq 0, \quad \bar{x} \in \mathcal{D}_c, \bar{x} \notin \mathcal{Z}_\delta,$$

$$V(\bar{x} + \bar{f}_d(\bar{x})) \leq V(\bar{x}), \quad \bar{x} \in \mathcal{D}_c, \bar{x} \in \mathcal{Z}_\delta. \quad (5)$$

Let $\mathcal{R} \triangleq \{ \bar{x} \in \mathcal{D}_c : \bar{x} \notin \mathcal{Z}_\delta, \mathcal{V}'(\bar{x})\bar{f}_c(\bar{x}) = 0 \} \cup \{ \bar{x} \in \mathcal{D}_c : \bar{x} \in \mathcal{Z}_\delta, V(\bar{x} + \bar{f}_d(\bar{x})) = V(\bar{x}) \}$ and let $\mathcal{M}$ denote the largest invariant set contained in $\mathcal{R}$. If $\bar{x}_0 \in \mathcal{D}_c$, then $\bar{x}(t) \to \mathcal{M}$ as $t \to \infty$. Finally, if $\mathcal{D} = \mathbb{R}^n$ and $V(\bar{x}) \to \infty$ as $\|\bar{x}\| \to \infty$, then all solutions $\bar{x}(t), t \geq 0$, of (3) and (4) that are bounded approach $\mathcal{M}$ as $t \to \infty$ for all $\bar{x}_0 \in \mathbb{R}^n$.

3. Hybrid adaptive stabilization for nonlinear hybrid dynamical systems using neural networks

In this section, we consider the problem of neural hybrid adaptive stabilization for nonlinear uncertain hybrid systems. Specifically, we consider the controlled state-dependent impulsive dynamical system (1) and (2) with $\mathcal{D} = \mathbb{R}^n$, $\mathcal{U}_c = \mathbb{R}^{m_c}$, and $\mathcal{U}_d = \mathbb{R}^{m_d}$, where $f_c : \mathbb{R}^n \to \mathbb{R}^n$ and $f_d : \mathbb{R}^n \to \mathbb{R}^n$ are continuously differentiable and satisfy $f_c(0) = 0$ and $f_d(0) = 0$, and $G_c : \mathbb{R}^n \to \mathbb{R}^{n \times m_c}$ and $G_d : \mathbb{R}^n \to \mathbb{R}^{n \times m_d}$.

In this paper, we assume that $f_c(\cdot)$ and $f_d(\cdot)$ are unknown functions, and $f_c(\cdot)$, $G_c(\cdot)$, $f_d(\cdot)$, and $G_d(\cdot)$ are given by

$$f_c(x) = A_c x + \Delta f_c(x), \quad G_c(x) = B_c G_{cn}(x),$$

$$f_d(x) = (A_d - I_n)x + \Delta f_d(x), \quad G_d(x) = B_d G_{dn}(x), \quad (7)$$

where $A_c \in \mathbb{R}^{n \times n}$, $A_d \in \mathbb{R}^{n \times n}$, $B_c \in \mathbb{R}^{n \times m_c}$, and $B_d \in \mathbb{R}^{n \times m_d}$ are known matrices, $G_{cn} : \mathbb{R}^n \to \mathbb{R}^{m_c \times n}$ and $G_{dn} : \mathbb{R}^n \to \mathbb{R}^{m_d \times n}$ are known matrix functions such that $\det G_{cn}(x) \neq 0$, $x \in \mathbb{R}^n$, and $\det G_{dn}(x) \neq 0$, $x \in \mathbb{R}^n$, and $\Delta f_c : \mathbb{R}^n \to \mathbb{R}^n$ and $\Delta f_d : \mathbb{R}^n \to \mathbb{R}^n$ are unknown functions belonging to the uncertainty sets $\mathcal{F}_c$ and $\mathcal{F}_d$, respectively, given by

$$\mathcal{F}_c = \{ \Delta f_c : \mathbb{R}^n \to \mathbb{R}^n : \Delta f_c(0) = 0, \Delta f_c(x) = B_c \delta_c(x), x \in \mathbb{R}^n \},$$

$$\mathcal{F}_d = \{ \Delta f_d : \mathbb{R}^n \to \mathbb{R}^n : \Delta f_d(0) = 0, \Delta f_d(x) = B_d \delta_d(x), x \in \mathbb{R}^n \}. \quad (9)$$

where $\delta_c : \mathbb{R}^n \to \mathbb{R}^{m_c}$ and $\delta_d : \mathbb{R}^n \to \mathbb{R}^{m_d}$ are uncertain continuously differentiable functions such that $\delta_c(0) = 0$ and $\delta_d(0) = 0$. It is important to note that since $\delta_c(x)$ and $\delta_d(x)$ are continuously differentiable and $\delta_c(0) = 0$ and $\delta_d(0) = 0$, it follows that there exist continuous matrix functions $\Delta_c : \mathbb{R}^n \to \mathbb{R}^{m_c \times n}$ and $\Delta_d : \mathbb{R}^n \to \mathbb{R}^{m_d \times n}$ such that $\delta_c(x) = \Delta_c(x)x$, $x \in \mathbb{R}^n$, and $\delta_d(x) = \Delta_d(x)x$, $x \in \mathbb{R}^n$. Furthermore, we assume that the continuous matrix functions $\Delta_c(\cdot)$ and $\Delta_d(\cdot)$ can be approximated over a compact set $\mathcal{D}_c \subset \mathbb{R}^n$ by a linear in the parameters neural
network up to a desired accuracy so that

\[ \text{col}_i(\Delta_c(x)) = W^T_{ci} \sigma_c(x) + \epsilon_{ci}(x), \quad x \in \mathcal{D}_c, \ i = 1, \ldots, n, \]  
\[ \text{col}_i(\Delta_d(x)) = W^T_{di} \sigma_d(x) + \epsilon_{di}(x), \quad x \in \mathcal{D}_d, \ i = 1, \ldots, n, \]  

where \( \text{col}_i(\Delta(\cdot)) \) denotes the \( i \)-th column of the matrix \( \Delta(\cdot) \), \( W^T_{ci} \in \mathbb{R}^{m_c \times s_c} \) and \( W^T_{di} \in \mathbb{R}^{m_d \times s_d} \), \( i = 1, \ldots, n \), are optimal unknown (constant) weights that minimize the approximation error over \( \mathcal{D}_c \), \( \epsilon_{ci} : \mathbb{R}^n \to \mathbb{R}^{m_c} \) and \( \epsilon_{di} : \mathbb{R}^n \to \mathbb{R}^{m_d} \), \( i = 1, \ldots, n \), are modeling errors such that \( \sigma_{\max}(\Upsilon_c(x)) \leq \gamma_c^{-1} \) and \( \sigma_{\max}(\Upsilon_d(x)) \leq \gamma_d^{-1} \), \( x \in \mathbb{R}^n \), where \( \Upsilon_c(x) \triangleq [\epsilon_{c1}(x), \ldots, \epsilon_{cn}(x)] \), \( \Upsilon_d(x) \triangleq [\epsilon_{d1}(x), \ldots, \epsilon_{dn}(x)] \), and \( \gamma_c, \gamma_d > 0 \), and \( \sigma_c : \mathbb{R}^n \to \mathbb{R}^{s_c} \) and \( \sigma_d : \mathbb{R}^n \to \mathbb{R}^{s_d} \) are given basis functions such that each component of \( \sigma_c(\cdot) \) and \( \sigma_d(\cdot) \) takes values between 0 and 1.

Next, defining

\[ \varphi_c(x) \triangleq \delta_c(x) - W^T_{c} [x \otimes \sigma_c(x)], \]  
\[ \varphi_d(x) \triangleq \delta_d(x) - W^T_{d} [x \otimes \sigma_d(x)], \]

where \( W^T_c \triangleq [W^T_{c1}, \ldots, W^T_{cs_c}] \in \mathbb{R}^{m_c \times s_c} \), \( W^T_d \triangleq [W^T_{d1}, \ldots, W^T_{ds_d}] \in \mathbb{R}^{m_d \times s_d} \), and \( \otimes \) denotes the Kronecker product, it follows from (11) and (12), and the Cauchy–Schwarz inequality that

\[ \varphi_j^T(x) \varphi_j(x) = \| \Delta_j(x)x - W^T_j [x \otimes \sigma_j(x)] \|^2 \]
\[ = \| \Delta_j(x)x - \Sigma_j(x)x \|^2 \]
\[ = \| \Upsilon_j(x)x \|^2 \]
\[ \leq \gamma_j^{-2} x^T x, \quad x \in \mathcal{D}_c, \quad j = c, d, \]  

where \( \| \cdot \| \) denotes the Euclidean norm and \( \Sigma_j(x) \triangleq [W^T_j \sigma_j(x), \ldots, W^T_{jn} \sigma_j(x)] \), \( j = c, d \). This corresponds to a nonlinear small gain-type norm bounded uncertainty characterization for \( \varphi_j(\cdot) \), \( j = c, d \) (see Fig. 3.1).

**Theorem 3.1.** Consider the nonlinear uncertain hybrid dynamical system \( G \) given by (1) and (2) where \( f_c(\cdot), G_c(\cdot), f_d(\cdot), \) and \( G_d(\cdot) \) are given by (7) and (8), and \( \Delta f_c : \mathbb{R}^n \to \mathbb{R}^n \) and \( \Delta f_d : \mathbb{R}^n \to \mathbb{R}^n \) belong to the uncertainty sets \( \mathcal{F}_c \) and \( \mathcal{F}_d \), respectively. For given \( \gamma_c, \gamma_d > 0 \), assume there exists a positive-definite matrix \( P \in \mathbb{R}^{n \times n} \) such that

\[ 0 = A^T_{cs} P + PA_{cs} + \gamma_c^{-2} PB_c B_c^T P + I_n + R_c, \]
\[ P = A^T_{sd} P A_{sd} - A^T_{sd} P B_d (B_d^T P B_d)^{-1} B_d^T P A_{sd} + (\alpha + \beta) I_n + R_d, \]

where \( A_{cs} \triangleq A_c + B_c K_c, \quad K_c \in \mathbb{R}^{m_c \times n}, \quad R_c \in \mathbb{R}^{n \times n} \) and \( R_d \in \mathbb{R}^{n \times n} \) are positive definite, \( \alpha > 0 \), and \( \beta \) satisfies

\[ \beta \geq \gamma_d^{-2} \left( \lambda_{\max}(B_d^T P B_d) + a \frac{1 + x^T P x}{c + [x \otimes \sigma_d(x)]^T [x \otimes \sigma_d(x)]} \right), \quad x \in \mathcal{Z}_x, \]
where

\[
a = \max \{c, n/\lambda_{\min}(P)\} \lambda_{\max}\left( B_d^T P B_d \left( I_m + \frac{1}{\sigma_d} B_d^T P B_d \right) \right) \tag{19}
\]

and \( c > 0 \). Finally, let \( A_{ds} \triangleq A_d + B_d K_d \), where \( K_d \triangleq -(B_d^T P B_d)^{-1} B_d^T P A_d \), and let \( Q_c \in \mathbb{R}^{m_c \times m_c} \) and \( Y \in \mathbb{R}^{ns_c \times ns_c} \) be positive definite. Then the neural hybrid adaptive feedback control law

\[
u_c(t) = G_c^{-1}(x(t)) \left[ K_c x(t) - \hat{W}_c^T(t)[x(t) \otimes \sigma_c(x(t))] \right], \quad x(t) \notin Z_x,
\]

\[
u_d(t) = G_d^{-1}(x(t)) \left[ K_d x(t) - \hat{W}_d^T(t)[x(t) \otimes \sigma_d(x(t))] \right], \quad x(t) \in Z_x,
\]

where \( \hat{W}_c^T(t) \in \mathbb{R}^{m_c \times ns_c}, t \geq 0, \hat{W}_d^T(t) \in \mathbb{R}^{m_d \times ns_d}, t \geq 0, \) and \( \sigma_c : \mathbb{R}^n \to \mathbb{R}^{n_c} \) and \( \sigma_d : \mathbb{R}^n \to \mathbb{R}^{n_d} \) are given basis functions, with update laws

\[
\dot{\hat{W}}_c^T(t) = \frac{1}{1 + x(t)^T P x(t)} Q_c B_c^T P x(t)[x(t) \otimes \sigma_c(x(t))]^T Y, \quad \hat{W}_c^T(0) = \hat{W}_{c0}^T, \quad x(t) \notin Z_x,
\]

\[
\Delta \hat{W}_c^T(t) = 0, \quad x(t) \in Z_x,
\]

\[
\dot{\hat{W}}_d^T(t) = 0, \quad \hat{W}_d^T(0) = \hat{W}_{d0}^T, \quad x(t) \notin Z_x,
\]

\[
\Delta \hat{W}_d^T(t) = \frac{1}{c + [x(t) \otimes \sigma_d(x(t))]^T [x(t) \otimes \sigma_d(x(t))] B_d^T t (x(t)^+ - A_{ds} x(t))[x(t) \otimes \sigma_d(x(t))]^T, \quad x(t) \in Z_x,
\]

where \( \Delta \hat{W}_c^T(t) \triangleq \hat{W}_c^T(t^+) - \hat{W}_c^T(t) \) and \( \Delta \hat{W}_d^T(t) \triangleq \hat{W}_d^T(t^+) - \hat{W}_d^T(t) \), guarantees that there exists a positively invariant set \( D_a \subset \mathbb{R}^n \times \mathbb{R}^{m_c \times n_c} \times \mathbb{R}^{m_d \times n_d} \) such that \((0, \hat{W}_c^T, \hat{W}_d^T) \in D_a\), where \( \hat{W}_c^T \in \mathbb{R}^{m_c \times n_c} \) and \( \hat{W}_d^T \in \mathbb{R}^{m_d \times n_d} \), and the solution \((x(t), \hat{W}_c^T(t), \hat{W}_d^T(t)) \equiv (0, W_c^T, W_d^T)\) of the closed-loop system given by (1), (2) and (20)–(25) is Lyapunov stable and \( x(t) \to 0 \) as \( t \to \infty \) for all \( \Delta f_c(\cdot) \in F_c, \Delta f_d(\cdot) \in F_d, \) and \((x_0, \hat{W}_{c0}^T, \hat{W}_{d0}^T) \in D_a\).

**Proof.** First, note that

\[
A_{ds}^T P B_d B_d^T P A_{ds} = (A_d + B_d K_d)^T P B_d B_d^T P (A_d + B_d K_d)
\]

\[
= (A_d - B_d (B_d^T P B_d)^{-1} B_d^T P A_d)^T P B_d B_d^T P (A_d - B_d (B_d^T P B_d)^{-1} B_d^T P A_d)
\]

\[
= 0,
\]

and hence, since \( A_{ds}^T P B_d B_d^T P A_{ds} \) is nonnegative definite, \( A_{ds}^T P B_d = 0 \). Furthermore, note that

\[
P = A_{ds}^T P A_{ds} + (\alpha + \beta) I_n + R_d.
\]

Now, with \( u_c(t), t \geq 0, \) and \( u_d(t_k), k \in N \), given by (20) and (21), respectively, it follows from (7) and (8) that the closed-loop hybrid system (1) and (2) is given by

\[
\dot{x}(t) = f_c(x(t)) + B_c \left[ K_c x(t) - \hat{W}_c^T(t)[x(t) \otimes \sigma_c(x(t))] \right], \quad x(0) = x_0, \quad x(t) \notin Z_x,
\]

\[
\Delta x(t) = f_d(x(t)) + B_d \left[ K_d x(t) - \hat{W}_d^T(t)[x(t) \otimes \sigma_d(x(t))] \right], \quad x(t) \in Z_x,
\]

or, equivalently, using (11) and (12),

\[
\dot{x}(t) = A_c x(t) + B_c \left[ \varphi_c(x(t)) - \hat{W}_c^T(t)[x(t) \otimes \sigma_c(x(t))] \right], \quad x(0) = x_0, \quad x(t) \notin Z_x,
\]

\[
\Delta x(t) = (A_d - I_n)x(t) + B_d \left[ \varphi_d(x(t)) - \hat{W}_d^T(t)[x(t) \otimes \sigma_d(x(t))] \right], \quad x(t) \in Z_x,
\]
where \( \hat{W}_d^T(t) = \hat{W}_c^T(t) - W_d^T \) and \( \hat{W}_d^T(t) = \hat{W}_d^T(t) - W_d^T \). Furthermore, define \( \hat{\sigma}_d(x) = x \otimes \sigma_d(x) \) and note that adding and subtracting \( W_d^T \) to and from \( 25 \) and using \( 31 \) it follows that

\[
\hat{W}_d^T(t^+) = \hat{W}_d^T(t) + \frac{1}{c + \hat{\sigma}_d(x(t))\hat{\sigma}_d(x(t))} B_d^T \left[ B_d[\varphi_d(x(t)) - \hat{W}_d^T(t)\hat{\sigma}_d(x(t))] \right] \left[ x(t) \otimes \sigma_d(x(t)) \right]^T
\]

\[
= \hat{W}_d^T(t) + \frac{1}{c + \hat{\sigma}_d(x(t))\hat{\sigma}_d(x(t))} [\varphi_d(x(t)) - \hat{W}_d^T(t)\hat{\sigma}_d(x(t))]\hat{\sigma}_d^T(x(t)), \quad x(t) \in \mathcal{Z}_x.
\]  

(32)

To show Lyapunov stability of the closed-loop hybrid system \((22)-(24)\) and \((30)-(32)\), consider the Lyapunov function candidate

\[
V(x, \hat{W}_c^T, \hat{W}_d^T) = \ln(1 + x^T P x) + \text{tr} \ Q_c^{-1} \hat{W}_c^T Y^{-1} \hat{W}_c + \text{tr} \hat{W}_d \hat{W}_d^T.
\]  

(33)

Note that \( V(0, W_c^T, W_d^T) = 0 \) and, since \( P, Q_c, \) and \( Y \) are positive definite and \( a > 0 \), \( V(x, \hat{W}_c^T, \hat{W}_d^T) > 0 \) for all \( (x, \hat{W}_c^T, \hat{W}_d^T) \neq (0, W_c^T, W_d^T) \). In addition, \( V(x, \hat{W}_c^T, \hat{W}_d^T) \) is radially unbounded. Now, letting \( x(t) \) denote the solution to \((30)\) and using \((22)\) and \((24)\), it follows that the Lyapunov derivative along the closed-loop system trajectories over the time interval \( t \in (t_k, t_{k+1}) \), \( k \in \mathcal{N} \), is given by

\[
\dot{V}(x(t), \hat{W}_c^T(t), \hat{W}_d^T(t)) = \frac{2x^T(t)P}{1 + x^T(t)Px(t)} \left[ A_{cs} x(t) + B_c \left[ \varphi_c(x(t)) - \hat{W}_c^T(t) [x(t) \otimes \sigma_c(x(t))] \right] \right]
\]

\[
+ 2 \text{tr} \ Q_c^{-1} \hat{W}_c^T(t) Y^{-1} \hat{W}_c(t)
\]

\[
\leq -x^T(t)(R_c + \gamma^{-2} B B_c^T P + I_n)x(t)
\]

\[
+ 2x^T(t) P B_c \left[ \varphi_c(x(t)) - \hat{W}_c^T(t) [x(t) \otimes \sigma_c(x(t))] \right]
\]

\[
+ 2 \text{tr} \hat{W}_c^T(t) \left( B_c^T P x(t) [x(t) \otimes \sigma_c(x(t))]^T \right)
\]

\[
= -x^T(t) R_c x(t) - x^T(t)(\gamma^{-2} B B_c^T P + I_n)x(t) + 2x^T(t) P B_c \varphi_c(x(t))
\]

\[
\leq -x^T(t) R_c x(t) - [\gamma^{-1} B B_c^T P x(t) - \gamma \varphi_c(x(t))]^T [\gamma^{-1} B B_c^T P x(t) - \gamma \varphi_c(x(t))]
\]

\[
\leq -x^T(t) R_c x(t)
\]

\[
\leq 0, \quad t_k < t < t_{k+1}.
\]  

(34)

Next, using \((23), (27)\) and \((32)\), the Lyapunov difference along the closed-loop system trajectories at the resetting times \( t_k, k \in \mathcal{N} \), is given by

\[
\Delta V(x(t_k), \hat{W}_c^T(t_k), \hat{W}_d^T(t_k)) = V(x(t_k^+), \hat{W}_c^T(t_k^+), \hat{W}_d^T(t_k^+)) - V(x(t_k), \hat{W}_c^T(t_k), \hat{W}_d^T(t_k))
\]

\[
= \ln \left( 1 + \left[ A_{ds} x(t_k) + B_d[\varphi_d(x(t_k)) - \hat{W}_d^T(t_k) [x(t_k) \otimes \sigma_d(x(t_k))] \right] \right)
\]

\[
\cdot P \left[ A_{ds} x(t_k) + B_d[\varphi_d(x(t_k)) - \hat{W}_d^T(t_k) [x(t_k) \otimes \sigma_d(x(t_k))] \right]
\]

\[
+ \text{tr} \left( \hat{W}_d^T(t_k) + \frac{1}{c + \hat{\sigma}_d(x(t_k))\hat{\sigma}_d(x(t_k))} \left[ \varphi_d(x(t_k)) - \hat{W}_d^T(t_k)\hat{\sigma}_d(x(t_k)) \right] \hat{\sigma}_d^T(x(t_k)) \right)^T
\]

\[
\cdot \left( \hat{W}_d^T(t_k) + \frac{1}{c + \hat{\sigma}_d(x(t_k))\hat{\sigma}_d(x(t_k))} \left[ \varphi_d(x(t_k)) - \hat{W}_d^T(t_k)\hat{\sigma}_d(x(t_k)) \right] \hat{\sigma}_d^T(x(t_k)) \right)
\]

\[
= \ln \left( 1 + \left[ x^T(t_k) A_{ds} P A_{ds} x(t_k) + 2x^T(t_k) A_{ds} P B_d \varphi_d(x(t_k)) \right.ight.
\]

\[
- 2x^T(t_k) A_{ds} P B_d \hat{W}_d^T(t_k)\hat{\sigma}_d(x(t_k)) + \varphi_d^T(x(t_k)) B_d P B_d \varphi_d(x(t_k))
\]

\[
- 2 \varphi_d^T(x(t_k)) B_d P B_d \hat{W}_d^T(t_k)\hat{\sigma}_d(x(t_k)) + \hat{\sigma}_d^T(x(t_k)) \hat{W}_d(t_k) B_d P B_d \hat{W}_d^T(t_k)\hat{\sigma}_d(x(t_k))
\]

\[
\left. \left. - 2 \varphi_d^T(x(t_k)) B_d P B_d \hat{W}_d^T(t_k)\hat{\sigma}_d(x(t_k)) + \hat{\sigma}_d^T(x(t_k)) \hat{W}_d(t_k) B_d P B_d \hat{W}_d^T(t_k)\hat{\sigma}_d(x(t_k)) \right) \right)
\]
Furthermore, note that
\[
\left[ -x^T(t_k)((\alpha + \beta)I_a + R_d)x(t_k) + \phi_d^T(x(t_k))B_d^T P B_d \phi_d(x(t_k)) - 2\phi_d^T(x(t_k))B_d^T P B_d \hat{W}_d(t_k)\phi_d(x(t_k)) \\
+ \tilde{\sigma}_d^T(x(t_k))\hat{W}_d(t_k)B_d^T P B_d \hat{W}_d(t_k)\tilde{\sigma}_d(x(t_k)) \right] \leq \left[ 1 + x^T(t_k)P x(t_k) \right]^{-1}
\]

where in (35) we used \( \ln a - \ln b = \ln \frac{a}{b} \) and \( \ln(1 + d) \leq d \) for \( a, b > 0, \) and \( d > -1, \) respectively, and \( \frac{\tilde{\sigma}_d^T \phi_d}{c + \tilde{\sigma}_d^T \phi_d} < 1. \)

Furthermore, note that \( \tilde{\sigma}_d^T \phi_d(x) \leq nx^T x. \)

Now, defining \( \Theta \triangleq \frac{1}{\nu_T} (B_d^T P B_d)^2, \) it follows from (35) that

\[
\Delta V(x(t_k), \hat{W}_c^T(t_k), \hat{W}_d^T(t_k)) \leq \left[ -x^T(t_k)R_d x(t_k) - \beta x^T(t_k)x(t_k) - \alpha [x^T(t_k)x(t_k) - \gamma_d^2 \phi_d^T(x(t_k)) \phi_d(x(t_k))] \right] \\
- \left[ \phi_d^T(x(t_k)), \tilde{\sigma}_d^T(x(t_k)) \hat{W}_d(t_k) \right] \left[ \frac{\alpha \gamma_d^2 I_a}{B_d^T P B_d} \Theta \right] \left[ \frac{\phi_d^T(x(t_k))}{\hat{W}_d(t_k)\tilde{\sigma}_d(x(t_k))} \right] \left[ 1 + x^T(t_k)P x(t_k) \right]^{-1}
\]

where

\[
\left\{ \begin{array}{l}
\frac{\tilde{\sigma}_d^T(x(t_k))}{c + \tilde{\sigma}_d^T(x(t_k))} \phi_d(x(t_k)) \\
\frac{\tilde{\sigma}_d^T(x(t_k))}{c + \tilde{\sigma}_d^T(x(t_k))} \phi_d(x(t_k)) \\
\frac{\tilde{\sigma}_d^T(x(t_k))}{c + \tilde{\sigma}_d^T(x(t_k))} \phi_d(x(t_k)) \\
\end{array} \right.
\]

(36)
\[ \check{R}_{d1}(x) \triangleq a(1 + x^TPx)I_m - (B_d^TPB_d + \Theta)(c + \check{\sigma}_d(x)\check{\sigma}_d(x)) \]
\[ \geq a(1 + x^TPx)I_m - B_d^TPB_d\left(I_m + \frac{1}{\alpha \gamma_d^2}B_d^TPB_d\right)(c + nx^Tx) \]
\[ \geq 0, \quad x \in D_c, \] (37)
and
\[ \check{R}_{d2}(x) \triangleq \beta \gamma_d^2(c + \check{\sigma}_d(x)\check{\sigma}_d(x))I_m - B_d^TPB_d(c + \check{\sigma}_d(x)\check{\sigma}_d(x)) - a(1 + x^TPx)I_m \]
\[ \geq (c + \check{\sigma}_d(x)\check{\sigma}_d(x))\left(\beta \gamma_d^2 - \lambda_{\text{max}}(B_d^TPB_d) - a\frac{1 + x^TPx}{c + \check{\sigma}_d(x)\check{\sigma}_d(x)}\right)I_m \]
\[ \geq 0, \quad x \in D_c. \] (38)

Hence, the Lyapunov difference given by (36) yields
\[ \Delta V(x(t_k), \hat{W}_c^T(t_k), \hat{W}_d^T(t_k)) \leq -\frac{x^T(t_k)R_d x(t_k)}{1 + x^T(t_k)P x(t_k)} - \frac{\check{\sigma}_d(x(t_k))\check{\sigma}_d(x(t_k))}{(c + \check{\sigma}_d(x)\check{\sigma}_d(x))(1 + x^T(t_k)P x(t_k))} \]
\[ \leq \frac{-x^T(t_k)R_d x(t_k)}{1 + x^T(t_k)P x(t_k)} \leq 0, \quad k \in N. \] (39)

Next, let
\[ \tilde{D}_a \triangleq \left\{(x, \hat{W}_c^T, \hat{W}_d^T) \in \mathbb{R}^n \times \mathbb{R}^{m_c \times n_c} \times \mathbb{R}^{m_d \times n_d} : V(x, \hat{W}_c^T, \hat{W}_d^T) \leq \alpha \right\}, \] (40)
where \( \alpha \) is the maximum value such that \( \tilde{D}_a \subseteq D_c \times \mathbb{R}^{m_c \times n_c} \times \mathbb{R}^{m_d \times n_d} \). Since \( \Delta V(x(t_k), \hat{W}_c^T(t_k), \hat{W}_d^T(t_k)) \leq 0 \) for all \( (x(t_k), \hat{W}_c^T(t_k), \hat{W}_d^T(t_k)) \in \tilde{D}_a \) and \( k \in N \), it follows that \( \tilde{D}_a \) is positively invariant. Next, since \( \tilde{D}_a \) is positively invariant, it follows that
\[ D_a \triangleq \left\{(x, \hat{W}_c^T, \hat{W}_d^T) \in \mathbb{R}^n \times \mathbb{R}^{m_c \times n_c} \times \mathbb{R}^{m_d \times n_d} : (x, \hat{W}_c^T - W_c^T, \hat{W}_d^T - W_d^T) \in \tilde{D}_a \right\} \] (41)
is also positively invariant.

Remark 3.1. Note that the conditions in Theorem 3.1 imply partial asymptotic stability, that is, the solution \( (x(t), \hat{W}_c^T(t), \hat{W}_d^T(t)) \equiv (0, W_c^T, W_d^T) \) to (22)–(24), (30) and (32) is Lyapunov stable. Furthermore, since \( R_c > 0 \) and \( R_d > 0 \), it follows from Theorem 2.1, with \( \mathcal{R} = \mathcal{M} = \{x, \hat{W}_c^T, \hat{W}_d^T \in \mathbb{R}^n \times \mathbb{R}^{m_c \times n_c} \times \mathbb{R}^{m_d \times n_d} : x = 0\} \), that \( x(t) \to 0 \) as \( t \to \infty \) for all \( x_0 \in \mathbb{R}^n \).  \( \square \)

Remark 3.2. Since the Lyapunov function used in the proof of Theorem 3.1 is a class \( K_\infty \) function, in the case where the neural network approximation holds in \( \mathbb{R}^n \), the control law (20) and (21) ensures global asymptotic stability with respect to \( x \). However, the existence of a global neural network approximator for an uncertain nonlinear map cannot in general be established. Hence, as is common in the neural network literature, for a given arbitrarily large compact set \( D_c \subset \mathbb{R}^n \), we assume that there exists an approximator for the unknown nonlinear map up to a desired accuracy (in the sense of (11) and (12)). In the case where \( \Delta_c(\cdot) \) and \( \Delta_d(\cdot) \) are continuous on \( \mathbb{R}^n \), it follows from the Stone-Weierstrass theorem that \( \Delta_c(\cdot) \) and \( \Delta_d(\cdot) \) can be approximated over an arbitrarily large compact set \( D_c \). In this case, our neuroadaptive hybrid controller guarantees semiglobal partial asymptotic stability.

Remark 3.3. Note that the neuroadaptive hybrid controller (20) and (21) can be constructed to guarantee partial asymptotic stability using standard linear \( H_\infty \) theory. Specifically, it follows from standard continuous-time \( H_\infty \) theory [22] that \( \|G_c(s)\|_{\infty} < \gamma_c \), where \( G(s) = E_c(sI_n - A_c)^{-1}B_c \) and \( E_c \) is such that \( E_c^TE_c = I_n + R_c \), if and only
if there exists a positive-definite matrix $P$ satisfying the bounded real Riccati equation (16). It is important to note that $\gamma_c > 0$ and $\gamma_d > 0$, which characterize the approximation error (13) and (14), respectively, over $D_c$, can be made arbitrarily large provided that we take a large number of basis functions in the parameterization of the uncertainty $\Delta_c(\cdot)$ and $\Delta_d(\cdot)$. In this case, noting that \[ \frac{1}{c + [x^T \sigma_d(x)]^T [x^T \sigma_d(x)]} \] in (18) is a bounded positive function, it can be shown that there always exist $\alpha$ and $\beta$ such that the conditions (16)–(19) are satisfied.

It is important to note that the hybrid adaptive control law (20)–(25) does not require explicit knowledge of the optimal weighting matrices $W_c$, $W_d$, and the positive constants $\alpha$ and $\beta$. Theorem 3.1 simply requires the existence of $W_c$, $W_d$, $\alpha$, and $\beta$ such that (16) and (17) hold. Furthermore, no specific structure on the nonlinear dynamics $f_c(x)$ and $f_d(x)$ is required to apply Theorem 3.1 other than the assumption that $f_c(x)$ and $f_d(x)$ are continuously differentiable and that the approximation error of the uncertain system nonlinearities lie in a small gain-type norm bounded conic sector. Finally, in the case where the pair $(A_d, B_d)$ is in controllable canonical form and $R_d$ in (17) is diagonal, it follows that $A_{ds} = \begin{bmatrix} A_0 & 0 \\ 0 & \alpha_d \end{bmatrix}$, where $A_0 \in \mathbb{R}^{(n-m_d) \times n}$ is a known matrix of zeros and ones capturing the multivariable controllable canonical form representation [23], and hence, the update law (25) is simplified as

$$ \Delta \hat{W}_d^T(t) = \frac{1}{c + [x(t) \otimes \sigma_d(x(t))]^T [x(t) \otimes \sigma_d(x(t))]^T} B_{d1}^T \Delta x(t) [x(t) \otimes \sigma_d(x(t))]^T, \quad x(t) \in Z_x, $$

(42)

since $B_{d1}^T A_{ds} = 0$.

4. Illustrative numerical example

In this section, we present a numerical example to demonstrate the utility of the proposed neural hybrid adaptive control framework for hybrid adaptive stabilization. Specifically, consider the nonlinear uncertain hybrid controlled system given by (1) and (2) with $n = 2$, $x = [x_1, x_2]^T$,

$$ f_c(x) = \begin{bmatrix} x_2 \\ \hat{f}_c(x) \end{bmatrix}, \quad G_c(x) = \begin{bmatrix} 0 \\ b_c \end{bmatrix}, \quad f_d(x) = \begin{bmatrix} -x_1 + x_2 \\ \hat{f}_d(x) \end{bmatrix}, \quad G_d(x) = \begin{bmatrix} 0 \\ b_d \end{bmatrix}, $$

(43)

where $\hat{f}_c : \mathbb{R}^2 \to \mathbb{R}$ and $\hat{f}_d : \mathbb{R}^2 \to \mathbb{R}$ are unknown, continuously differentiable functions. Furthermore, assume that the resetting set $Z_x$ is given by

$$ Z_x = \{ x \in \mathbb{R}^2 : \mathcal{X}(x) = 0, x_2 > 0 \}, $$

(44)

where $\mathcal{X} : \mathbb{R}^2 \to \mathbb{R}$ is a continuously differentiable function given by $\mathcal{X}(x) = x_1$. Here, we assume that $f_c(x)$ and $f_d(x)$ are unknown and can be written in the form of (7) and (8) with

$$ A_c = A_d = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, $$

$$ \Delta f_c(x) = [0, \hat{f}_c(x)]^T, \quad \Delta f_d(x) = [0, \hat{f}_d(x)]^T, \quad B_c = [0, b_c]^T, \quad B_d = [0, b_d]^T, \quad G_{cn}(x) = G_{dn}(x) = 1. $$

We assume that $\Delta f_c(x)$ and $\Delta f_d(x)$ are unknown and can be written as $\Delta f_c(x) = B_c \delta_c(x)$ and $\Delta f_d(x) = B_d \delta_d(x)$, where $\delta_c(x) = \frac{1}{b_c} \hat{f}_c(x)$ and $\delta_d(x) = \frac{1}{b_d} \hat{f}_d(x)$.

Next, let $K_c = \frac{1}{b_c} [k_{c1}, k_{c2}]$ and $K_d = \frac{1}{b_d} [k_{d1}, k_{d2}]$, where $k_{c1}, k_{c2}, k_{d1},$ and $k_{d2}$ are arbitrary scalars, such that

$$ A_{cs} = A_c + B_c K_c = \begin{bmatrix} 0 & 1 \\ k_{c1} & k_{c2} \end{bmatrix}, $$

$$ A_{ds} = A_d + B_d K_d = \begin{bmatrix} 0 & 1 \\ k_{d1} & k_{d2} \end{bmatrix}. $$

Now, with the proper choice of $k_{c1}, k_{c2}, k_{d1},$ and $k_{d2}$, it follows from Theorem 3.1 that if there exists $P > 0$ satisfying (16) and (17), then the neural hybrid adaptive feedback controller (20) and (21) guarantees $x(t) \to 0$ as $t \to \infty$. Specifically, here we choose $k_{c1} = -1, k_{c2} = -1, k_{d1} = -0.2, k_{d2} = -0.5, \gamma_c = 10, \gamma_d = 20, b_c = 3, b_d = 1.4, c = 1, \alpha = 1, \sigma_d(x) = [\tanh(0.1 x_2), \ldots, \tanh(0.6 x_2)]^T,$ and
Fig. 4.1. Phase portraits of uncontrolled and controlled hybrid system.

Fig. 4.2. State trajectories versus time.

\[ R_c = \begin{bmatrix} 2.6947 & 2.4323 \\ 2.4323 & 5.8019 \end{bmatrix}, \quad R_d = \begin{bmatrix} 8.0196 & 2.0334 \\ 2.0334 & 1.0569 \end{bmatrix}, \quad (45) \]

so that \( P \) satisfying (16) and (17) is given by

\[ P = \begin{bmatrix} 10.0196 & 2.0334 \\ 2.0334 & 12.7523 \end{bmatrix}. \]

With \( \hat{f}_c(x) = -a_1x_1 - a_2(x_1^2 - a_3)x_2, \hat{f}_d(x) = -x_2 - a_4x_1^2 - a_5\frac{x_2^3}{1+x_2^2} - a_6x_2^3, a_1 = 1, a_2 = 2, a_3 = 1, a_4 = -5, a_5 = -2, a_6 = 8, Y = 0.02I_3, \sigma_c(x) = \begin{bmatrix} \frac{1}{1+e^{-x_1}}, \ldots, \frac{1}{1+e^{-x_1}}, \frac{1}{1+e^{-x_2}}, \ldots, \frac{1}{1+e^{-x_2}} \end{bmatrix}, \]

and initial conditions
$x(0) = [1, 1]^T$, $\hat{W}_c^T(0) = 0_{1 \times 6}$, and $\hat{W}_d^T(0) = 0_{1 \times 6}$. Fig. 4.1 shows the phase portraits of the uncontrolled and controlled hybrid system. Figs. 4.2 and 4.3 show the state trajectories versus time and the control signals versus time, respectively. Finally, Fig. 4.4 shows the adaptive gain history versus time.
5. Conclusion

A direct hybrid neuroadaptive nonlinear control framework for hybrid nonlinear uncertain dynamical systems was developed. Using Lyapunov methods the proposed framework was shown to guarantee partial asymptotic stability of the closed-loop hybrid system; that is, asymptotic stability with respect to part of the closed-loop system states associated with the hybrid plant dynamics. In the case where the nonlinear hybrid system is represented in normal form, the nonlinear hybrid adaptive controller was constructed without requiring knowledge of the system dynamics. Finally, a numerical example was presented to show the utility of the proposed hybrid adaptive stabilization scheme.

Acknowledgments

This research was supported in part by the Japan Science and Technology Agency under CREST program and the Air Force Office of Scientific Research under Grant FA9550-06-1-0240.

References